

CLAIMS

1. A polarimetric system for analyzing a sample comprising :
 - an excitation section (1) emitting a light beam, said excitation section (1) comprising a polarization state generator (4) containing a polarizer (5) linearly polarizing the incident light beam (2) along a direction of polarization (i),
 - an analysis section (3) comprising a polarization state detector (8) containing an analyzer (9), and detection means (10),
- 5 10 - a processing unit (11), wherein,
 - the polarization state generator (4) (PSG) and the polarization state detector (8) (PSD) comprise each a first and a second liquid crystal elements (13, 14) LC_j ($j=1, 2$) having, for each LC_j element (13) of the PSG (respectively for each LC'_j element (14) of the PSD), an extraordinary axis making an angle θ_j (resp. θ'_j) with respect to the direction of polarization (i) and a retardation δ_j (resp. δ'_j) between its ordinary and extraordinary axes, said liquid crystal (LC'_j) elements (14) being positioned in reverse order in the PSD with respect to the LC_j elements (13) of the PSG, and
- 15 20 - the orientation angles θ'_j are equal to θ_j ($j=1,2$) and the retardations δ'_j are equal to $-\delta_j$ ($j=1,2$), (modulo 2π).
- 25 30 2. A polarimetric system according to claim 1, wherein said liquid crystal elements (13, 14) LC_j ($j=1, 2$) are nematic (NLCs) liquid crystals and the polarimetric system comprises an electronic control unit enabling polarization modulation by varying the retardations δ_j and δ'_j for NLCs.
3. A polarimetric system according to claim 1 wherein said liquid crystal elements (13, 14) LC_j ($j=1,2$) are ferroelectric (FLCs) liquid crystals, and the polarimetric system comprises an electronic control enabling polarization modulation by varying the orientation angles θ_j and θ'_j for FLCs.
4. A polarimetric system according to claims 1 and 2, wherein:
 - the couple of retardations (δ_1, δ_2) is varied in the following sequence (Δ_1, Δ_1), (Δ_1, Δ_2), (Δ_2, Δ_1), (Δ_2, Δ_2), where Δ_1 and Δ_2 verify the formulae ($\Delta_1 = 315^\circ + p 90^\circ$) and ($\Delta_2 = 135^\circ + p 90^\circ$) respectively,

where p is the same integer in both formulae, with a tolerance of +/- 20°,

- the orientations angles θ_1 and θ_2 verify the formulae ($\theta_1 = \varepsilon 27^\circ + q 90^\circ$) and ($\theta_2 = \varepsilon 72^\circ + r 90^\circ$) respectively where $\varepsilon = \pm 1$ has the same value in both equations while q and r are any integer, with a tolerance of +/-10°.

- 5 5. A polarimetric system according to claims 1 and 3, wherein:
- the orientations of the extraordinary axes are set sequentially to (θ_1, θ_2) , $(\theta_1+45^\circ, \theta_2)$, $(\theta_1, \theta_2+45^\circ)$, $(\theta_1+45^\circ, \theta_2+45^\circ)$,
- 10 10. the retardations (δ_1, δ_2) verify $\delta_1=80^\circ+/-15^\circ$ and $\delta_2=160^\circ+/-15^\circ$, while the orientation angles (θ_1, θ_2) are given by $\theta_1 = 67^\circ+/-10^\circ$ and $\theta_2 = 160^\circ+/-40^\circ$.
- 15 6. A polarimetric system according to claim 5, wherein the polarimetric system is suitable for a range of wavelengths, and a fixed retardation plate (17, 18) is placed between said two FLCs, both in the PSG and in the PSD.
- 20 7. A polarimetric system according to claim 6, wherein said polarimetric system is optimized for the spectral range from 420 nm to 800 nm, and the retardation plate is a quartz plate and PSG comprises
- a linear polarizer, set at an orientation angle $\theta=0$,
 - a first ferroelectric liquid crystal, with a retardation $\delta_1=90^\circ+/-5^\circ$ at 510 nm, set at an orientation angle $\theta_1=-10^\circ+/-5^\circ$,
 - a quartz plate, providing a retardation $\delta_Q=90^\circ+/-5^\circ$ at 633 nm, set at an orientation angle $\theta_Q=5^\circ+/-5^\circ$,
- 25 25. a second ferroelectric liquid crystal, with a retardation $\delta_2=180^\circ+/-15^\circ$ at 510 nm, set at an orientation angle $\theta_2=71^\circ+/-10^\circ$.
- 30 8. A polarimetric system according to claim 1 to 7 wherein said polarimetric system is an ellipsometer.
- 35 9. A polarimetric system according to any one of the claims 1 to 7, wherein said polarimetric system is a Mueller polarimetric system for analyzing a sample (7) through the measurement of the sixteen coefficients of its Mueller matrix.
10. A polarimetric system according to any one of claims 1 to 9 wherein the light beam (2) emitted by the excitation section (1) is in

the spectral range 400-1500 nm for nematic liquid crystals, and 420-800 nm for ferroelectric liquid crystals.

5 11 A polarimetric system according to any one of claims 1 to 10 wherein the excitation section (1) comprises a monochromator positioned before the polarization state generator (4) (PSG).

12. A polarimetric system according to any one of claims 1 to 10 wherein the detection means (10) comprises a monochromator, placed after the PSD.

10 13. A polarimetric system according to any one of claims 1 to 12 wherein the detection means (10) comprises a multipoint photosensitive detector, adapted with the processing unit (11) to polarimetric imaging.

14. A polarimetric system according to claim 13 wherein the multipoint photosensitive detector is a charge coupled detector (CCD).

15 15. A calibration process of a polarimetric system involving measurement of at least a reference sample (7) in which

- one illuminates the sample (7) with a polarized incident light beam (2) emitted by a polarisation state generator (PSG) containing a polarizer, said PSG modulating the light beam (2) polarization,
- said sample (7) transmits or reflects a measurement beam,
- one detects the measurement beam with an analysis section (3) comprising a polarization state detector (8) (PSD) containing an analyzer (9), and detection means (10), and
- one processes the electrical signals produced by the detection means (10) with a processing unit (11),

wherein,

- Said PSG contains a first and a second liquid crystal elements (13) LC_j (j=1,2) positioned after the polarizer, said LC_j elements (13) having retardations δ_j between their ordinary and extraordinary axes and said extraordinary axes making angles θ_j with respect to the polarization direction defined by the linear polarizer so that by varying the retardation δ_j of each LC_j element for a fixed value of the θ_j angle, when the LC_j elements are nematic LCs, or by switching the orientation angle θ_j when the LC_j elements are ferroelectric LCs, one modulates the incident light

beam (2) polarization, the PSG having a modulation matrix (**W**) that is non singular,

- Said PSD contains a third and a fourth liquid crystal elements (14) LC'_j (j=1,2) positioned before the analyser, said LC'_j elements (14) being the same as the LC_j elements of the PSG but positioned in the reverse order, so that by varying the retardation δ'_j of each element for fixed values of θ'_j angles when the LC'_j are nematic LCs, or by switching the values of angles θ'_j for fixed δ'_j when the LC'_j are ferroelectric LCs, one generates a detection matrix (**A**) for the analysis section (3), said matrix being non singular and so that for a given set of retardations (δ_j , δ'_j) (j=1,2), or for a given set of orientation angles (θ_j , θ'_j), one produces a measured quantity (D_n) and so that the processing unit (11) produces the raw data matrix $\mathbf{B} = \mathbf{A}\mathbf{M}\mathbf{W}$, where (**M**) is the Mueller matrix of the sample (7).

15 16. A calibration process according to claim 15, said calibration process being adapted to ellipsometric measurements in transmission of samples (7) assumed to be dichroïc retarders (DR), in order to determine their ellipsometric parameters (τ , Ψ , Δ) wherein the process comprises taking a complete measurement of a reference sample (7) consisting of a DR defined by a Mueller matrix (**M**₀) with known parameters τ_0 , Ψ_0 and Δ_0 , said reference sample (7) being propagation in air and (**M**₀) then being the identity matrix (**I**₀), said measurement providing a reference raw data matrix $\mathbf{B}_0 = \mathbf{A}\mathbf{M}_0\mathbf{W}$.

25 17. A calibration process according to claim 15, said calibration process being adapted to ellipsometric measurements in reflection of samples (7) assumed to be dichroïc retarders (DR) in order to determine their ellipsometric parameter (τ , ψ , Δ) wherein the process comprises taking a complete measurement of a reference sample (7) consisting of a DR defined by a Mueller matrix (**M**₀) with known parameters (τ_0 , ψ_0 , Δ_0), said sample (7) being a metallic mirror or a known sample (7) for a system working in reflection mode, said measurement providing a reference raw data matrix $\mathbf{B}_0 = \mathbf{A}\mathbf{M}_0\mathbf{W}$.

30 18. A calibration process according to claim 15, said process being adapted to the complete Mueller polarimetry of any sample (7) in transmission, wherein the calibration process comprises:

- choosing a set of reference samples (7) elements (p) comprising dichroic retarders with approximately known Mueller matrices (M_p), defined by the parameters (τ_p, Ψ_p, Δ_p) one of these elements being the identity matrix (I_o) describing propagation in air,
- 5 - for each of the reference samples (p), taking a complete measurement of said sample (7), set at an orientation angle θ_p , by modulating the incoming light polarization and analyzing the outgoing light polarization, constructing the matrix $(AR(-\theta_p)M_pR(\theta_p)W)$ using the processing unit, this matrix being a product of the detection matrix (A), the Mueller matrix ($R(-\theta_p)M_pR(\theta_p)$) of said element p set at the angle θ_p , with $R(\theta)$ a matrix describing a rotation by an angle θ about the z axis and the modulation matrix (W),
- 10 - calculating the product $(AI_oW)^{-1}(AR(-\theta_p)M_pR(\theta_p)W)$ for each reference sample (7) p in order to obtain an experimental matrix (C_p),
- 15 - determining the actual values of $(\tau_p, \Psi_p, \Delta_p)$, and thus the matrix M_p , independently of the angles θ_p , from the eigenvalues of (C_p),
- constructing a matrix ($K_{tot}(\theta_p)$) equal to $\sum_p (H_p(\theta_p)^T H_p(\theta_p))$ where the matrix $H_p(\theta_p)$ is defined as $H_p(\theta_p)[X] = R(-\theta_p)M_pR(\theta_p)X - XC_p$ where (X) is any real 4x4 matrix,
- 20 - determining the eigenvalues λ_i ($i = 1$ to 16) of the ($K_{tot}(\theta_p)$) matrix in order to extract the modulation matrix (W) that verifies $K_{tot}(W)=0$, the p reference samples (7) being chosen so that one and only one eigenvalue λ_i vanishes when the angles (θ_p) used in the calculation of $K_{tot}(\theta_p)$ are set equal to their actual values during the calibration measurements, while
- 25 - the other eigenvalues λ_j , being sorted in decreasing order of value, verify $Z=\lambda_{15}/\lambda_1 < 1$ and the ratio Z is maximised,
- determining the angles (θ_p) by requiring one of the eigenvalues $K_{tot}(\theta_p)$ to vanish, W being the associated eigenvector,
- 30 - determining the detection matrix (A) by constructing the product $(AI_oW)(W^{-1})$.
19. A calibration process according to claim 18 wherein a set of reference samples (7) comprises
- a linear polarizer set at $\theta_1=0^\circ$ orientation,
 - a linear polarizer set at $\theta_2 = 90^\circ +/- 5^\circ$ orientation,
- 35 - a retardation plate with a retardation $\delta=110^\circ +/- 30^\circ$ set at $\theta_3=30^\circ +/- 5^\circ$.

20. A calibration process according to claim 19, wherein the retardation plate is an achromatic quarterwave plate.

21. A calibration process according to claim 15, said calibration process being adapted to the complete Mueller polarimetry of a sample
5 (7) in reflection:

- choosing a set of reference samples (7) comprising a linear polarizer, defined by its Mueller matrix \mathbf{M}_{pol} , and a first DR1 and a second DR2 dichroic retarders, said DR_i having Mueller matrices (\mathbf{M}_i), with i=(1, 2) respectively, with approximately known values of the parameters τ_i , Ψ_i , Δ_i ,

10 - with each of the following sequence of elements, taking a measurement by modulating the incoming light polarization and analyzing the outcoming light polarization, the origin of the azimuthal angles ($\theta=0$) being taken in the plane of incidence,

◆ DR₁ alone, set at $\theta=0$, yielding a measured matrix $\mathbf{B}_1=\mathbf{A}\mathbf{M}_1\mathbf{W}$

15 ◆ DR₂ alone, set at $\theta=0$, yielding a measured matrix $\mathbf{B}_2=\mathbf{A}\mathbf{M}_2\mathbf{W}$

◆ DR₁, set at $\theta=0$, and preceded by the polarizer (5) set at an orientation angle θ_1 , yielding a measured matrix $\mathbf{B}_{p1}=\mathbf{A}\mathbf{M}_1\mathbf{R}(-\theta_1)\mathbf{M}_{\text{pol}}\mathbf{R}(\theta_1)\mathbf{W}$, where $\mathbf{R}(\theta)$ is a matrix describing a rotation by an angle θ about the z axis

20 ◆ DR₁, set at $\theta=0$, and followed by the polarizer, set at an orientation angle θ_2 , yielding the measured matrix $\mathbf{B}_{p2}=\mathbf{A}\mathbf{R}(-\theta_2)\mathbf{M}_{\text{pol}}\mathbf{R}(\theta_2)\mathbf{M}_1\mathbf{W}$,

- Calculating the products $\mathbf{C}_1=\mathbf{B}_2^{-1}\mathbf{B}_1$ and $\mathbf{C}_2=\mathbf{B}_1\mathbf{B}_2^{-1}$ and then the matrices $\mathbf{N}_1=\mathbf{M}_2^{-1}\mathbf{M}_1$ and $\mathbf{N}_2=\mathbf{M}_1\mathbf{M}_2^{-1}$ through their eigenvalues, which are the same as those of \mathbf{C}_1 and \mathbf{C}_2 ,

25 - Calculating the products $\mathbf{C}_{p1}=\mathbf{B}_2^{-1}\mathbf{B}_{p1}=\mathbf{W}^{-1}\mathbf{N}_1\mathbf{R}(-\theta_1)\mathbf{M}_{\text{pol}}\mathbf{R}(\theta_1)\mathbf{W}$ and $\mathbf{C}_{p2}=\mathbf{B}_{p2}\mathbf{B}_2^{-1}=\mathbf{A}\mathbf{R}(-\theta_2)\mathbf{M}_{\text{pol}}\mathbf{R}(\theta_2)\mathbf{N}_2\mathbf{A}^{-1}$,

- Defining a \mathbf{K}_1 matrix as $\mathbf{K}_1(\theta_1)[\mathbf{X}]=\mathbf{H}_1^T\mathbf{H}_1 + \mathbf{H}_{p1}(\theta_1)^T\mathbf{H}_{p1}(\theta_1)$, where for any any 4x4 real matrix \mathbf{X} , $\mathbf{H}_1[\mathbf{X}]$ and $\mathbf{H}_{p1}(\theta_1)[\mathbf{X}]$ are defined as

$$\mathbf{H}_1[\mathbf{X}]=\mathbf{N}_1\mathbf{X}-\mathbf{X}\mathbf{C}_1 \text{ and } \mathbf{H}_{p1}(\theta_1)[\mathbf{X}]=\mathbf{N}_1\mathbf{R}(-\theta_1)\mathbf{M}_{\text{pol}}\mathbf{R}(\theta_1)\mathbf{X}-\mathbf{X}\mathbf{C}_{p1}$$

30 - Determining the modulation matrix \mathbf{W} and the orientation θ_1 by requiring that $\mathbf{K}_1(\theta_1)$ has one vanishing eigenvalue, and \mathbf{W} is the eigenvector associated with this vanishing eigenvalue,

- Determining similarly the analysis matrix \mathbf{A} as the eigenvector associated with the unique vanishing eigenvalue of the matrix

$$35 \quad \mathbf{K}_2(\theta_2)[\mathbf{X}]=\mathbf{H}_2^T\mathbf{H}_2 + \mathbf{H}_{p2}(\theta_2)^T\mathbf{H}_{p2}(\theta_2)$$

where for any real 4x4 matrix \mathbf{X}

$$\mathbf{H}_2[\mathbf{X}] = \mathbf{C}_2\mathbf{X} - \mathbf{X}\mathbf{N}_2, \quad \mathbf{H}_{p2}(\theta_2)[\mathbf{X}] = \mathbf{C}_{p2}\mathbf{X} - \mathbf{X}\mathbf{R}(-\theta_2)\mathbf{M}_{pol}\mathbf{R}(\theta_2)\mathbf{N}_2$$

-reference samples (7) are then chosen according to the following criteria:

- 5 ○ the 16x16 real symmetrical matrices $\mathbf{K}_1(\theta_1)$ and $\mathbf{K}_2(\theta_2)$ will only have one vanishing eigenvalue, if and only if the angles θ_1 and θ_2 used for their evaluation are equal to the azimuthal angles of the polarizers during the calibration measurements,

- 10 ○ The next eigenvalues are as large as possible, or, more precisely, the ratios $Z = \lambda_{15}/\lambda_1$ of the smallest nonvanishing eigenvalues (λ_{15}) over the largest (λ_1) eigenvalues of \mathbf{K}_1 and \mathbf{K}_2 are as large as possible.

22. A calibration process according to claim 21, wherein the set of reference samples (7)

- 15 - a linear polarizer set at $\theta_1=45^\circ +/- 5^\circ$
 - a linear polarizer set at $\theta_2=-45^\circ +/- 5^\circ$, and
 - a couple of samples (7) equivalent to a first DR₁ and a second DR₂ dichroic retarders, both oriented at $\theta=0$ with respect to the incidence plane, with Mueller matrices \mathbf{M}_1 and \mathbf{M}_2 such that the products $\mathbf{M}_2^{-1}\mathbf{M}_1$ and $\mathbf{M}_2^{-1}\mathbf{M}_1$ are the Mueller matrices of a DR with $\Psi=45^\circ +/- 30^\circ$ and $\Delta=90^\circ +/- 10^\circ$.

23. A calibration process according to claim 22, wherein for spectroscopic applications, said reference samples (7) comprise a metallic mirror.

- 25 24. A calibration process according to claim 22, wherein for spectroscopic applications said reference samples (7) comprise an achromatic quarter-wave plate, oriented with one axis in the incidence plane placed before or after a metallic mirror.

- 30 25. A polarimetric measurement process intended for measurement of a sample (7) represented by the coefficients of a Mueller matrix (\mathbf{M}), in which:

- emitting an incident light beam (2) linearly polarised along a direction of polarisation(i),
- modulating the incident beam polarization,

- sending the modulated incident beam to the sample (7), and returning a measurement beam,
 - collecting the measurement beam through a polarisation analysis section (3),
- 5 • detecting the measurement beam after the polarization analysis section (3) and producing electrical signals forming the raw data matrix $\mathbf{B} = \mathbf{AMW}$,
- transmitting the electrical signals to a processing unit,
wherein
- 10 • modulating the incident beam polarization by means of two liquid crystal elements LC_j ($j=1, 2$) (13) by varying either the angular orientations θ_j , of the extraordinary axes with respect to the polarization direction (i) of the linear polarizer (5) when the liquid crystals (LCs) comprise ferroelectric LCs (FLCs), or the retardations δ_j at fixed
15 orientations when the LCs comprise nematic LCs (NLCs),
- producing measured quantities (D_n) by means of an analysis section (3) comprising two liquid crystal elements (14) LC'_j ($j=1, 2$) by varying the retardation δ'_j of each element for fixed values of θ'_j angles when the LCs are NLCs, or the values of the orientation angles θ'_j for fixed values of
20 retardation δ'_j ($j=1, 2$) when the LCs are FLCs,
 - producing by means of the processing unit the Mueller matrix ($M=A^{-1}B W^{-1}$) with its sixteen coefficients.

25 26. An ellipsometric measurement procedure providing the ellipsometric parameters (τ, Ψ, Δ) of a sample (7) to be analysed, said sample (7) being assumed to be a dichroic retarder (DR), wherein the procedure comprises

- collecting the raw data matrix \mathbf{B} according to claim 25
- calculating the matrix $\mathbf{C} = \mathbf{B}_0^{-1} \mathbf{B} = \mathbf{W}^{-1} \mathbf{M}_0^{-1} \mathbf{M} \mathbf{W}$, the eigenvalues of \mathbf{C} being the same as those of $\mathbf{M}_0^{-1} \mathbf{M}$, and two of them, λ_{R1} and λ_{R2}
30 being real positive, while the other two λ_{c1} and λ_{c2} are complex conjugates.
- deducing the ellipsometric parameters of the studied sample (7) from these eigenvalues and the known parameters τ_0 , Ψ_0 and Δ_0 of the calibration sample (7) using

$$\Psi = \arctan\left(\sqrt{\frac{\lambda_{R1}}{\lambda_{R2}}} \tan \Psi_0\right)$$

$$\Delta = \Delta_0 + \frac{1}{2} \operatorname{Arg}\left(\frac{\lambda_{C1}}{\lambda_{C2}}\right)$$

$$\tau = \frac{\tau_0 (\lambda_{R1} + \lambda_{R2}) \sin^2 2\Psi_0}{(1 - \cos 2\Psi_0 \cos 2\Psi)} ,$$

- 5 - testing the validity of the description of the sample (7) as a DR
 through the relationship

$$|\lambda_{C1}|^2 = |\lambda_{C2}|^2 = \lambda_{R1} \lambda_{R2}$$

- 10 which must be verified to within the measurement accuracy if the
 sample (7) to be analysed is a DR.